Extrapolation of truncated transfer functions for compensating time-domain measurements

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***Abstract*—**Three methods (M1-M3) are implemented to extrapolate the missing spectrum of a truncated cable transfer function. The accuracy of the extrapolation methods is tested by de-embedding the measurement of several time-domain pulses with a rise time between 100 to 1000 ps. Rise time and peak amplitude errors are calculated and compared with the incident pulses waveform parameters.

Keywords— Extrapolation, Hilbert transform, transfer function

I. INTRODUCTION

Subnanosecond HPEM sources produce fast pulses with spectral content reaching the GHz range. The measurement chain used to characterize such systems comprises numerous elements such as sensors, baluns, attenuators, and cables (see, e.g., Figure 1). Interpolation and extrapolation methods are usually required during signal processing to adequately compensate in the frequency domain since the measurement chain elements are generally characterized in separate frequency spans and different frequency steps.

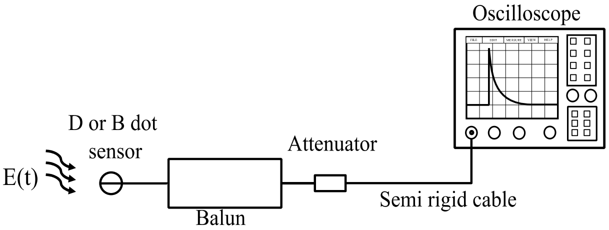

Figure 1. Simplified block diagram for time-domain measurements.

The real and imaginary parts of the complex transfer functions obtained after interpolation and extrapolation must satisfy the Kramers-Kronig relations. Previous authors have proposed reconstructing missing or corrupted data using these criteria. For example, a minimum phase reconstruction of a waveform from its spectral magnitude response was proposed in [1]. A comparison between the Cauchy method and the discrete Hilbert transform properties is proposed for interpolating and extrapolating a corrupted spectrum in [2]. The Hilbert transform integrals in the Z-domain were used to guarantee causality in [3].

II. MEASUREMENT SETUP

The measurement setup is composed of a D-Dot sensor, a balun, a variable attenuator, and a semi-rigid cable connected to a fast 40 GSa/s – 6GHz oscilloscope, as shown in Figure 1. The S-parameters of the semi-rigid cable were measured using a Vector Network Analyzer (VNA) between 100 kHz to 20 GHz in 10^4 points. The cable transfer function (TF) was truncated between 10 MHz and 3 GHz to test the accuracy of extrapolation algorithms.

Three extrapolation methods were implemented to reconstruct the truncated TF missing spectrum. M1 uses a constant extrapolation by keeping the last measured value of the truncated spectrum, while M2 uses 10 dB/dec attenuation to reconstruct the high-frequency spectrum. Finally, M3 uses the Hilbert transform in Z-domain [3]. The comparison between the measured and extrapolated TFs is shown in Figure 2.

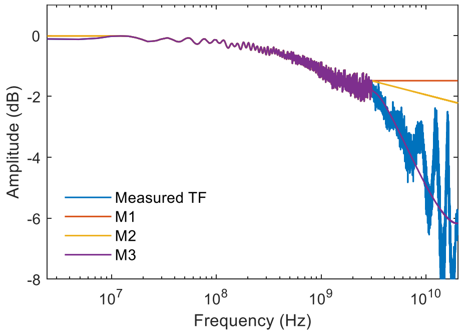

Figure 2. Extrapolated transfer functions of the semi-rigid cable.

III. RESULTS

Several incident EM pulses are simulated with a rise time between 100 ps to 1000 ps and 30 kV/m peak amplitude to test the accuracy of the extrapolation. The expected signal at the oscilloscope is calculated numerically using the measured cable transfer function. Then, the incident electric field is determined by compensating for the cable presence by using the reconstructed spectra with M1-M3. The D-Dot sensor, balun, and variable attenuator are assumed as ideal elements. The obtained waveform parameters (rise time and peak amplitude) are calculated and compared between the incident and compensated electric fields.

REFERENCES

[1] F. M. Tesche, “On the use of the Hilbert transform for processing measured CW data,” *IEEE Transactions on Electromagnetic Compatibility*, vol. 34, no. 3, pp. 259–266, 1992, DOI: 10.1109/15.155838.

[2] S. Narayana, T. K. Sarkar, R. Adve, M. Wicks, and V. Vannicola, “A Comparison of Two Techniques for the Interpolation/Extrapolation of Frequency Domain Responses,” *Digital Signal Processing*, vol. 6, no. 1, pp. 51–67, 1996, DOI: 10.1006/dspr.1996.0006.

[3] J. Becerra, F. Vega, and F. Rachidi, “Extrapolation of a Truncated Spectrum With Hilbert Transform for Obtaining Causal Impulse Responses,” *IEEE Transactions on Electromagnetic Compatibility*, vol. 59, no. 2, pp. 454–460, 2017, DOI: 10.1109/temc.2016.2613044.