The Regge Method for a Vertical Half-Circular Loop above Conducting Ground

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Abstract— The response of the thin-wire half - circular loop perpendicular to PEC ground to the lumped voltage source by application the Regge-Watson transformation is reduced to the sum over frequency-dependent complex values $m_v(k)$ $(v=1,2,3...)$. Corresponding curves $\{Re(m_v(k)), Im(m_v(k)\}$ on the complex m-plane describes deep physical properties of the scattering system.

Keywords- thin wires; circular loop; Regge theory

I. INTRODUCTION

Direct numerical methods, e.g., Method of Moments usually used to calculate EM field coupling with wiring do not allow deep research into the physical essence of the problem. This can only be achieved by using analytical or semi-analytical methods. The exact analytical solutions that are possible for structures with high geometrical symmetry are especially important: an infinite straight wire, a circular wire, a helix wire and their combinations that keep symmetry, e.g., an infinite straight wire over an PEC surface, half-circular loop perpendicular to PEC surface, twisted pairs, etc.

II. RESULTS

Here, we consider a circular half-loop perpendicular to the PEC ground. This structure is the only finite transmission line - like wiring structure for which there is an exact solution to the mixed-potential integral equations. This solution can be obtained by Fourier series for any type of excitations [1], including distributed excitations (e.g., by external plane wave) or lumped excitations (e.g., by voltage source). The solution for the lumped excitation is especially important because it is a Green's function for the current and yields the solution with arbitrary excitation.

To obtain this solution with appropriate accuracy, one has to use 100-400 terms in the Fourier series. In our previous paper [2], we have shown, how to simplify this Fourier solution and, using the phenomenological physical method, approximately obtained the main term of the current excited by lumped source. This current is analog of TEM mode excited by a lumped source in the infinite straight wire above a PEC ground. In this work, we use the Watson - Regge transformation and represent the Fourier sum as an integral in the complex plane of the parameter m, which is an integer in the classical Fourier solution. The integral is defined by the zeros of the modal impedance per-unit length in the complex plane of the parameter m, which zeros define the so called Regge poles, in analogue with scattering theory in quantum mechanics [3].

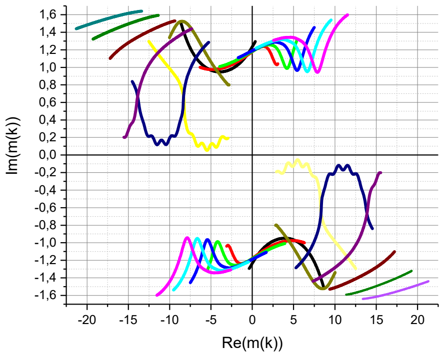

Figure 1. Regge trajectories (26 poles) for circular loop R=4m, r_0=1cm, 0.75 $m^{-1} \leq k \leq 3.875$ m^{-1}.

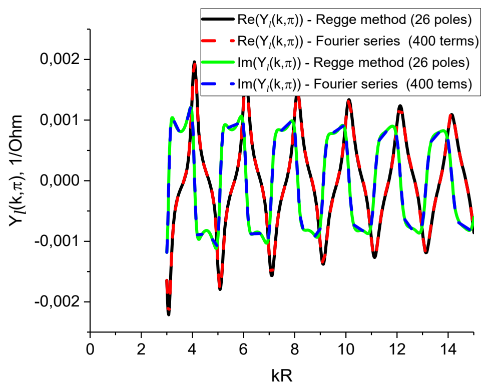

Figure 2. Frequency dependence of the admittance function for current for the circular loop (R=4m, r_0=1cm) calculated by Regge and Fourier methods. Voltage source and observation point are arranged at the points $\phi=0$ and $\phi=\pi$.

The positions of the poles on the complex plane depend on the frequency and form so called Regge trajectories (see Fig.1). The sum over the Regge poles is an exact solution of the problem and equals the sum of Fourier series (see Fig.2). The term corresponding to the pole with the smallest imaginary part coincides with the phenomenological solution. Moreover, after some manipulation on this term, one can obtain the SEM poles of the first layer for the wiring structure.

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